Follow the Laggard? – Not all Bottlenecks Are Created Equal
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Abstract. In a mixed push and pull system, the location of the slowest link does not generally coincide with where the greatest congestion is. Goldratt's solution to the ever-lengthening line of hikers thus assumes away the most difficult problem of correctly identifying the slowest link in a system. By putting the slowest hiker in front, Goldratt's solution seems to imply that the exact location of the most restrictive bottleneck is critical to increasing the throughput rate. In fact, the location of the slowest link does not affect the throughput rate of a batch process. This paper uses a system dynamics model to clarify some of the conceptual confusion that has been created by Goldratt's counterintuitive conclusions.

Keywords: Bottleneck, excess inventory, push production system, pull production system, just-in-time, Goldratt.

The Hikers' Story

A group of scouts was hiking in single file along a narrow trail that allowed no overtaking. They started out evenly spaced. As time went on, the distance between the first hiker and the last hiker got ever longer. The gap in front of Herbie, the slowest hiker who is in the middle of the troop was particularly wide. After the scoutmaster switched Herbie to the front, the line shortened and the gap between hikers also narrowed. When Herbie's hiking speed was increased by spreading his heavy load among the faster hikers, the whole troop arrived at their destination in good time. So goes the fascinating story made legendary by Goldratt's popular production management novel "The Goal." (1986: Chapters 13-15)

The lesson from this story is somewhat unclear though. Is the lesson simply that throughput can be increased by widening the most visible bottleneck? Or that throughput depends on where the bottleneck is located? Specifically, would the troop have arrived at their destination in the same amount of time if Herbie's speed had been increased to the same extent but without switching him to the front of the troop? And more generally, how can the most binding bottleneck in a production line be identified and does the location of the most binding bottleneck affect the amount and location of excess inventory in the production line? These questions will be examined in the context of a push vs pull production system.

Getting There – Three Separate Objectives

In the hikers' problem, two related but different objectives have to be achieved. One, the distance between the leader and the last hiker must be within some tolerable limit. This is necessary for troop safety and easy supervision by the scoutmaster. Two, the last hiker must complete the hike within an acceptable time limit. The troop is considered to have arrived at the destination as a group only when the last hiker completes the hike.

But a third objective may also be important. For overnight camping, the campsite has to be cleared of debris, tents have to be pitched, and a fire has to be started for cooking. If some faster hikers arrive at the destination ahead of the rest of the troop, they can use their lead time to do all these chores. If there is still time left over, they can stretch out and smell the wild flowers.

Plus, they can hike at a speed more suited to their natural ability without being constrained by the slow-moving Herbie.

Putting Herbie, the slowest hiker, at the head of the troop and lightening his load to increase his hiking speed will surely achieve the first two objectives: namely, keeping the troop close together and completing the hike sooner. But holding everybody back behind the slowest hiker surely won't achieve the third objective: namely, having an advance party to prepare the campsite before the rest of the troop arrives.

Follow the Laggard

The reason why putting Herbie, the slowest hiker, at the head of the troop can keep the troop closer together is that the faster hikers behind Herbie will bunch up behind him. Suppose Herbie can walk 200 feet per minute and the rest of the troop can all walk at 300 feet per minute. Because the faster hikers behind Herbie cannot overtake him, the fastest speed they can travel is no more than 200 feet per minute. Even if the followers slack off a bit from their average natural speed of 300 feet per minute, their slower natural speed (say at 270 feet per minute) will still be much faster than Herbie's 200 feet per minute. That means the gap between Herbie and the followers and the gaps among the followers will be reduced to zero.

The time it takes for the troop to arrive at their destination will be given by:
trip completion time in minutes = total distance in feet / 200 feet per minute.
In other words, the trip completion time is the same for Herbie as well as for the last hiker in line.

If the speed of Herbie is increased to 250 feet per minute, the trip completion time in minute is reduced to:

total distance in feet / 250 feet per minute.

Again, the trip completion time is the same for Herbie as well as for the last hiker in line.

Follow the Leader

Suppose Herbie is put at the end of the line and his speed is increased from 200 feet per minute to 250 feet per minute as before. Since Herbie's speed is not constrained by the faster speed of his fellow troopers in front of him, he can complete his trip in:

total distance in feet / 250 feet per minute.

In other words, in exactly the same amount of time as when he is put at the head of the troop. But the leader will have arrived 20% [i.e., (300 – 250) / 250] earlier than Herbie. And the total gap in space between the leader and Herbie will also increase by 20%. In other words, Herbie will be left far behind the rest of the troop, making supervision of the troop much more difficult.

Sandwiching the Laggard

If Herbie is sandwiched between a small advance party and the rest of the troop, the advance party will travel at 300 feet per minute and the rest of troop will travel at Herbie's improved speed (250 feet per minute). The small advance party will arrive at the destination in:

total distance in feet / 300 feet per minute.

The rest of the troop behind Herbie will arrive at the destination in:

total distance in feet / 250 feet per minute.

In other words, 20% later than the advance party. But in exactly the same amount of time as when Herbie is put at the head of the whole troop.

The advantage of sandwiching Herbie in such a configuration is quite clear. Namely, most of the troop is still kept close together for easy supervision. Yet, the faster advance party can prepare the campsite when it arrives at the destination earlier than the rest of the troop. The faster and preferably more experienced hikers in the advance party can pretty much take care of themselves without any supervision from the scoutmaster.

Push vs Pull

When the slowest hiker is leading the troop, the hike is analogous to a pull production (just-in-time) system. In a pull production system, the slowest link determines how fast the whole system operates. Because the pull downstream controls how much is produced upstream, no excess inventory will be produced. In the hiking problem, the speed of the whole troop is determined by the slowest hiker, Herbie. When Herbie is in front, no matter how fast the others can travel, their speed cannot exceed Herbie's speed. Because the slowest natural speed of the other hikers is faster than Herbie's , their default speed is Herbie's speed. That means all gaps between hikers behind Herbie will disappear. The disappearance of gaps between hikers is analogous to the non-existence of excess inventory in a pull production system. In this system, suppliers upstream will only produce and deliver as much as the downstream users require (Shingo 1981).

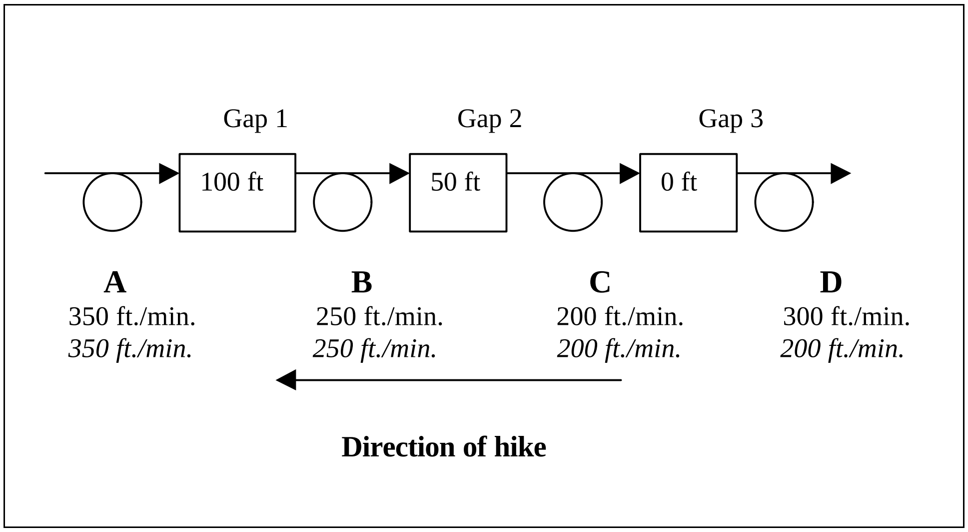
If the hikers are re-arranged in descending order of their natural speeds with the fastest hiker in front, the hike is analogous to a push production system. In a push production system, inflow from upstream is pushed through the downstream links regardless of their flow capacities. Similarly, the fastest hiker in front walks at his natural speed regardless of how slow the hikers behind him walks. The ever widening gaps between the faster and slower hikers are analogous to excess inventory built up at links along the production line wherever the inflow rate exceeds the outflow capacity.

But the speed of the whole troop is still determined by the slowest hiker. Before the slowest hiker completes the hike, the troop as a whole has not completed the hike. In the same way, the throughput of a push production system is not determined by how fast upstream inflow can be, but how fast the slowest link can complete its job.

Who Is Herbie? – Identifying Bottlenecks

When hikers of different natural speeds are arranged neither in ascending or descending order of their speeds, the identity of the slowest hiker is not intuitively obvious because there is a mixture of push and pull in the *actual* speeds of individual hikers. For example, if 4 hikers with different natural speeds are arranged in the following order: **A** at 350 feet per minute, **B** at 250 feet per minute, **C** at 200 feet per minute, and **D** at 300 feet per minute. The widest gap between hikers is not in front of **C**, the slowest hiker at 200 feet per minute. Instead, the widest gap is in front of **B**, the second slowest hiker at 250 feet per minute. This is so because gaps between hikers are determined by the difference between the speeds of two adjacent hikers if the hiker in front travels faster than the one behind. Sine the difference in speeds between **A** and **B** is 100 feet (i.e., 350 – 250) per minute while the difference between **B** and **C** is only 50 feet (i.e., 250-200) per minute, the widest gap appears in front of **B** (the second slowest hiker) instead of **C** (the slowest hiker). In other words, from **A** to **C**, a push system is at work with **A** pushing ahead regardless of how wide the gaps between him and the slower hikers behind him are. The gaps between hikers are analogous to excess inventory along a push production line (Figure 1).

***Figure 1*. Who Is the Slowest Hiker in a Push System?**



Notes: Gap distance indicates addition to the relevant gap per minute. Direction of arrows into or from gaps indicates addition to or subtraction from the relevant gap, not direction of the hike. Natural speeds are in regular font. *Actual* speeds are in *italics*.

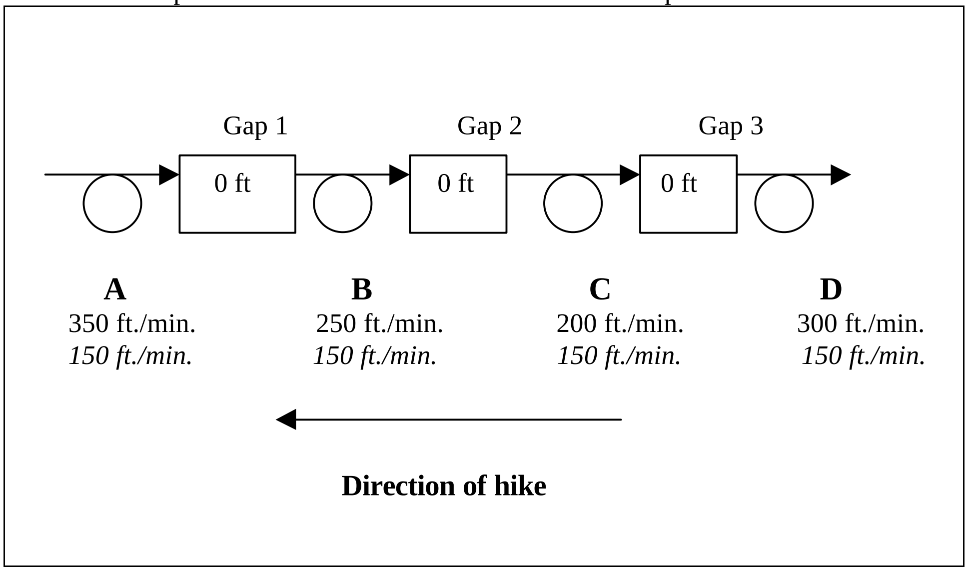
From **C** to **D**, the gap is zero because **C**'s (the slowest hiker at 200 feet per minute) natural and *actual* speed is slower than **D**'s natural speed (at 300 feet per minute). **D**'s *actual* speed is therefore reduced to **C**'s speed at 200 feet per minute. Thus, a pull system is at work between **C** and **D** with **D**'s *actual* speed constrained by how fast **C** can pull**.** The absence of a gap between them is analogous to the absence of excess inventory in a pull production system.

If the identity of the slowest hiker is associated with the widest gap in front of the hiker, then **B** (instead of **C**) will be mistakenly identified as the slowest hiker. But narrowing the gap between **A** and **B** by speeding up **B** will not shorten the arrival time of **D** because **D** cannot travel faster than **C** even if there is no gap between **A** and **B**. On the other hand, if **C**'s (the slowest hiker) speed is increased, **D**'s arrival time will surely be shortened.

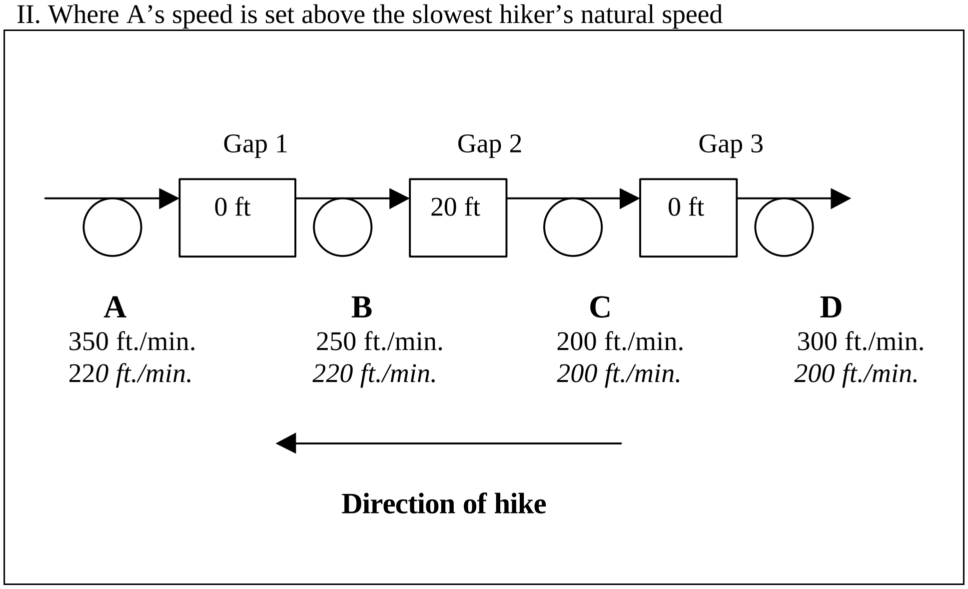
In a pure pull system, the slowest link is readily exposed when the system is sped up. For example, if **A**'s *actual* speed is set below the slowest hiker's natural speed at 150 feet per minute, **B**, **C**, and **D**'s *actual* speeds will also be 150 feet per minute. Since there is no gap between hikers, the throughput of the system is also 150 feet per minute (Figure 2.I).

Figure 2. **Who Is the Slowest Hiker in a Pull System?**

I. Where A's speed is set below the slowest hiker's natural speed



II. Where A's speed is set above the slowest hiker's natural speed



Notes: Gap distance indicates addition to the relevant gap per minute. Direction of arrows into or from gaps indicates addition to or subtraction from the relevant gap, not direction of the hike. Natural speeds are in regular font. *Actual* speeds are in *italics*.

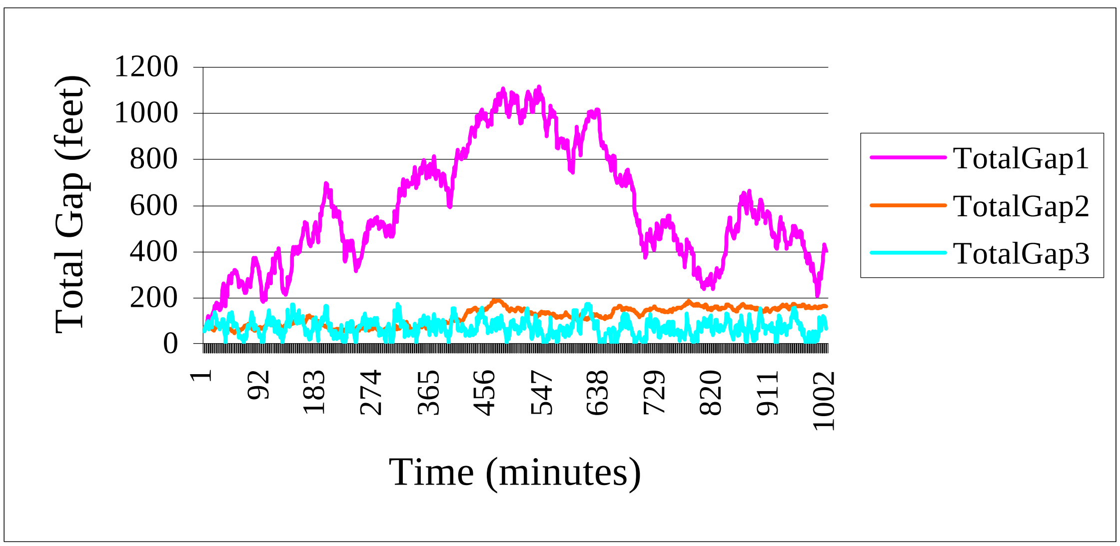
But when **A**'s *actual* speed is ramped up to above the natural speed of the slowest hiker (i.e., **C** at 200 feet per minute) to 220 feet per minute, a gap immediately appears in front of **C** exposing him as the slowest hiker (Figure 2.II). Because there is no confusion about the identity of the slowest link in a pull system, resources are less likely to be misallocated to increase the capacity of non-critical links.

Not All Gaps Are Created Equal Either

In the above example, hikers were assumed to maintain a constant speed. So all gaps between hikers are a result of differences in speeds between hikers. But the width of these gaps can widen considerably over time if the *actual* speeds deviate from the mean natural speeds and overtaking is not allowed, even when the mean natural speeds of the hikers are identical. Suppose Hiker1 in front happens to be walking a little faster than his mean speed in time1 while Hiker2 behind happens to be walking at or a little slower than his mean speed, the gap between Hiker1 and Hiker2 will widen. This gap can be reduced or eliminated when Hiker2 happens to be walking faster than Hiker1 in time2. But since Hiker2 is not allowed to overtake Hiker1, Hiker2 can never store up negative gap (to offset future positive gap) even when he is capable of walking ahead of Hiker1 in some time periods. So the average gap between Hiker1 and Hiker2 can increase considerably over time because of a combination of constraints (i.e., no overtaking) and statistical fluctuations around the mean natural speed (Goldratt, 1986: Chapters 11 and 13).

For example, when the natural speeds of 4 hikers with identical mean natural speeds (initially evenly spaced at 20 feet apart) are generated by a Poisson distribution and no overtaking is allowed, the total gap between the hikers can go above 1000 feet over a 1000-period simulation (see TotalGap 1 in Figure 3). The average total gap of 586 feet is almost 10 times as wide as the initial 60 feet. If overtaking were allowed, we would expect that the average total gap over time be close to 60 feet given identical mean natural speeds.

Figure 3. **Total Gap under Same Mean Speeds but Different Statistical Fluctuations.**



Notes: Initially, the 4 hikers are 20 feet apart from each other. For Total Gap1: the natural speeds of 4 hikers are generated by a Poisson distribution each with a mean speed of 300 feet per minute. No overtaking is allowed. For Total Gap2: the natural speeds of 4 hikers are generated by a uniform distribution each with a mean speed of 300 feet per minute and a ± 5 feet range above and below the mean speed. No overtaking is allowed. For Total Gap3: the natural speeds of 4 hikers are generated by a Poisson distribution each with a mean speed of 300 feet per minute. No overtaking is allowed. But actual speeds of follower hikers are adjusted to match the speed of the hiker in front when the gap between them exceeds 30 feet. Powersim model equations may be requested from kkfung@memphis.edu.

These wide gaps due to accumulated slowness can be considerably reduced if statistical fluctuations are reduced. For example, when statistical fluctuations around the identical mean natural speeds of 300 feet per minute are reduced to ± 5 feet per minute, the maximum total gap is less than 200 feet (with an average total gap of 118 feet) in a 1,000-minute simulation (see TotalGap 2 in Figure 3).

Of course, individual hikers' *actual* speeds do not independently fluctuate according to some statistical distribution. More likely, the speed of the hiker behind is affected by the speed of the hiker in front, if only to maintain an acceptable gap. Suppose the natural speeds of individual hikers (each with a mean speed of 300 feet per minute) are governed by a Poisson distribution but the *actual* speeds are modified to match the speed of the hiker immediately in front if the gap between them exceeds 30 feet. The resulting maximum total gap between hikers is reduced to less than 200 feet in a 1,000-minute simulation (see TotalGap 3 in Figure3). The average total gap of 71 feet is almost identical to the initial total gap of 60 feet.

Thus, when speed adaptation between hikers occurs, any glaringly wide gap must be a result of different mean natural speeds among hikers, and not merely a combined result of constraints (i.e., no overtaking) and statistical fluctuations around identical mean natural speeds.

References

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Shingo, Shigeo. 1981. *A Study of the Toyota Production System*. Cambridge, MA: Productivity Press.

Note

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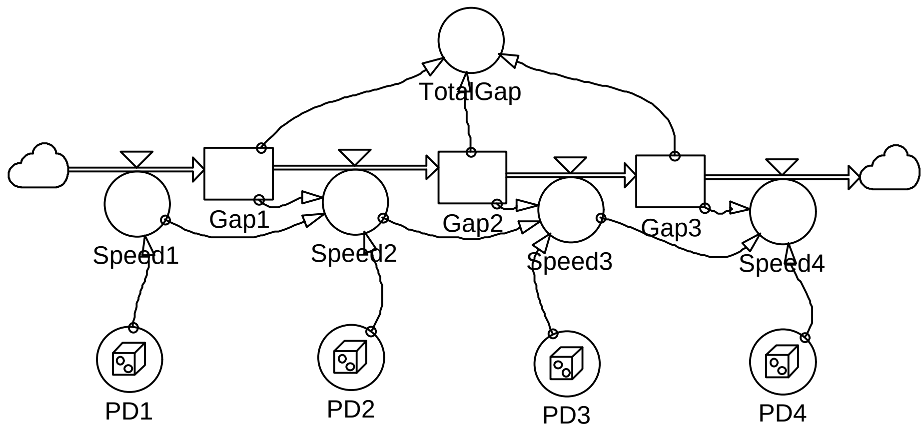
Powersim Model Diagram for Total Gap Models



Gaps are determined by the difference in speeds between two hikers. The speed of the hiker behind is constrained by the speed of the hiker in front plus the gap between them.

The 3 models have essentially the same diagram except for Total Gap Model 2, where PD_i's are renamed as $RANDOM_i$'s.

The mean speeds for all hikers are the same. The Poisson distributions (PD's) around the mean speeds are identical for Model 1 and Model 3. The RANDOM distribution of Model 2 is a uniform distribution with a narrower value range. Otherwise, it is how these distributions are modified by the Speed flow equations that determine the different Total Gap profiles in Figure 3.

Appendix 1: Powersim Equations for Total Gap 1 (!hike.sim)

init	Gap1 = 20
flow	Gap1 = -dt*Speed2 +dt*Speed1
unit	Gap1 = feet
init	Gap2 = 20
flow	Gap2 = -dt*Speed3 +dt*Speed2
unit	Gap2 = feet
init	Gap3 = 20
flow	Gap3 = -dt*Speed4 +dt*Speed3
unit	Gap3 = feet
aux	Speed1 = PD1
doc	Speed1 = Hiker1 at head of group can walk without any constraint at his natural speed which is generated by a Poisson distribution with mean=300 feet per minute.
unit	Speed1 = feet per minute
aux	Speed2 = IF(PD2>(Speed1+Gap1), Speed1+Gap1, PD2)
doc	Speed2 = Hiker2 who is behind Hiker1 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2. Otherwise, Hiker2 can only walk at a speed equal to the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2.
unit	Speed2 = feet per minute
aux	Speed3 = IF(PD3>(Speed2+Gap2), Speed2+Gap2, PD3)
doc	Speed3 = Hiker3 who is behind Hiker2 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker2 plus the gap between Hiker2 and Hiker3. Otherwise, Hiker3 can only walk at a speed equal to the actual speed of Hiker2 plus the gap between Hiker2 and Hiker3.
unit	Speed3 = feet per minute
aux	Speed4 = IF(PD4>(Speed3+Gap3), Speed3+Gap3, PD4)
doc	Speed4 = Hiker4 who is behind Hiker3 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker3 plus the gap between Hiker3 and Hiker4. Otherwise, Hiker4 can only walk at a speed equal to the actual speed of Hiker3 plus the gap between Hiker3 and Hiker4.
unit	Speed4 = feet per minute
aux	PD1 = POISSON(300, 0.001)
doc	PD1 = Natural speed of Hiker1 as generated by a Poisson distribution with mean=300 feet per minute. 0.001 is a seed variable to ensure the same set of random numbers is generated between simulation runs.
unit	PD1 = feet per minute
aux	PD2 = POISSON(300, 0.002)
doc	PD2 = Natural speed of Hiker2 as generated by a Poisson distribution with mean=300 feet per minute. 0.002 is a seed variable to ensure the same set of random numbers is generated between simulation runs.
unit	PD2 = feet per minute
aux	PD3 = POISSON(300, 0.003)
doc	PD3 = Natural speed of Hike31 as generated by a Poisson distribution with mean=300 feet per minute. 0.003 is a seed variable to ensure the same set of random numbers is generated between simulation runs.
unit	PD3 = feet per minute
aux	PD4 = POISSON(300, 0.004)
doc	PD4 = Natural speed of Hiker4 as generated by a Poisson distribution with mean=300 feet per minute. 0.004 is a seed variable to ensure the same set of random numbers is generated between simulation runs.
unit	PD4 = feet per minute
aux	TotalGap = Gap1+Gap2+Gap3
spec	dt = 1.00000
spec	method = Euler (fixed step)

Appendix2: Powersim Equations for Total Gap 2 (!hike2.sim)

init	Gap1 = 20
flow	Gap1 = -dt*Speed2 +dt*Speed1
unit	Gap1 = feet
init	Gap2 = 20
flow	Gap2 = -dt*Speed3 +dt*Speed2
unit	Gap2 = feet
init	Gap3 = 20
flow	Gap3 = -dt*Speed4 +dt*Speed3
unit	Gap3 = feet
aux	Speed1 = Random1
doc	Speed1 = Hiker1 at head of the group can walk without any constraint at his natural speed which is generated by a uniform distribution of integers between 295 and 305 feet per minute.
unit	Speed1 = feet per minute
aux	Speed2 = IF(Random2>(Speed1+Gap1), Speed1+Gap1, Random2)
doc	Speed2 = Hiker2 behind Hiker1 can walk at his natural speed (which is generated by a uniform distribution as an integer between 295 and 305 feet per minute) only if his natural speed is less than the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2. Otherwise, Hiker2 can only walk at a speed equal to the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2.
unit	Speed2 = feet per minute
aux	Speed3 = IF(Random3>(Speed2+Gap2), Speed2+Gap2, Random3)
doc	Speed3 = Hiker3 behind Hiker2 can walk at his natural speed (which is generated by a uniform distribution as an integer between 295 and 305 feet per minute) only if his natural speed is less than the actual speed of Hiker2 plus the gap between Hiker2 and Hiker3. Otherwise, Hiker3 can only walk at a speed equal to the actual speed of Hiker2 plus the gap between Hiker2 and Hike3.
unit	Speed3 = feet per minute
aux	Speed4 = IF(Random4>(Speed3+Gap3), Speed3+Gap3, Random4)
doc	Speed4 = Hiker4 behind Hiker3 can walk at his natural speed (which is generated by a uniform distribution as an integer between 295 and 305 feet per minute) only if his natural speed is less than the actual speed of Hiker3 plus the gap between Hiker3 and Hiker4. Otherwise, Hiker4 can only walk at a speed equal to the actual speed of Hiker3 plus the gap between Hiker3 and Hiker4.
unit	Speed4 = feet per minute
aux	Random1 = INT(RANDOM(295, 305, 0.001))
doc	Random1 = Natural speed of Hiker1 as generated by a uniform distribution integers between 295 feet per minute and 305 feet per minute. 0.001 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	Random1 = feet per minute
aux	Random2 = INT(RANDOM(295, 305, 0.002))
doc	Random2 = Natural speed of Hiker2 as generated by a uniform distribution of integers between 295 feet per minute and 305 feet per minute. 0.002 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	Random2 = feet per minute
aux	Random3 = INT(RANDOM(295, 305, 0.003))
doc	Random3 = Natural speed of Hiker3 as generated by a uniform distribution of integers between 295 feet per minute and 305 feet per minute. 0.003 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	Random3 = feet per minute
aux	Random4 = INT(RANDOM(295, 305, 0.004))
doc	Random4 = Natural speed of Hiker4 as generated by a uniform distribution as an integer between 295 feet per minute and 305 feet per minute. 0.004 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	Random4 = feet per minute
aux	TotalGap = Gap1+Gap2+Gap3
spec	dt = 1.00000
spec	method = Euler (fixed step)

Appendix 3: Powersim Equations for Total Gap 3 (!hikexx.sim)

init	Gap1 = 20
flow	Gap1 = -dt*Speed2 +dt*Speed1
unit	Gap1 = feet
init	Gap2 = 20
flow	Gap2 = -dt*Speed3 +dt*Speed2
unit	Gap2 = feet
init	Gap3 = 20
flow	Gap3 = -dt*Speed4 +dt*Speed3
unit	Gap3 = feet
aux	Speed1 = PD1
doc	Speed1 = Hiker1 at head of group can walk at his natural speed which is generated by a Poisson distribution with mean=300 feet per minute.
unit	Speed1 = feet per minute
aux	Speed2 = IF(PD2>=(Speed1+Gap1), Speed1+Gap1, IF((PD2<Speed1) AND Gap1 >30 , Speed1, PD2))
doc	Speed2 = Hiker2 who is behind Hiker1 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2, and the gap between Hiker1 and Hiker2 is equal to or less than 30 feet and not widening. If the gap is greater than 30 feet and widening, Hiker2 must increase his natural speed to match the actual speed of Hiker1. If Hiker2's natural speed is equal to or greater than the actual speed Hiker1 plus the gap between Hiker1 and Hiker2, Hiker2 can only walk at a speed equal to the actual speed of Hiker1 plus the gap between Hiker1 and Hiker2.
unit	Speed2 = feet per minute
aux	Speed3 = IF(PD3>=(Speed2+Gap2), Speed2+Gap2, IF((PD3<Speed2)AND Gap2 >30 , Speed2, PD3))
doc	Speed3 = Hiker3 who is behind Hiker2 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker2 plus the gap between Hiker2 and Hiker3, and the gap between Hiker2 and Hiker3 is equal to or less than 30 feet and not widening. If the gap is greater than 30 feet and widening, Hiker3 must increase his actual speed to match the actual speed of Hiker2. If Hiker3's natural speed is equal to or greater than the actual speed Hiker2 plus the gap between Hiker2 and Hiker3, Hiker3 can only walk at a speed equal to the actual speed of Hike2 plus the gap between Hiker2 and Hiker3.
unit	Speed3 = feet
aux	Speed4 = IF(PD4>=(Speed3+Gap3), Speed3+Gap3, IF((PD4<Speed3) AND Gap3 >30 , Speed3, PD4))
doc	Speed4 = Hiker4 who is behind Hiker3 can walk at his natural speed (which is generated by a Poisson distribution with mean=300 feet per minute) only if his natural speed is less than the actual speed of Hiker3 plus the gap between Hiker3 and Hiker4, and the gap between Hiker3 and Hiker4 is equal to or less than 30 feet and not widening. If the gap is greater than 30 feet and widening, Hiker4 must increase his actual speed to match the actual speed of Hiker3. If Hiker4's natural speed is equal to or greater than the actual speed Hiker3 plus the gap between Hiker3 and Hiker4, Hiker4 can only walk at a speed equal to the actual speed of Hike3 plus the gap between Hiker3 and Hiker4.
unit	Speed4 = feet per minute
aux	PD1 = POISSON(300, 0.001)
doc	PD1 = Natural speed of Hiker1 as generated by a Poisson distribution with mean = 300 feet per minute. 0.001 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	PD1 = feet per minute
aux	PD2 = POISSON(300, 0.002)

doc	PD2 = Natural speed of Hiker2 as generated by a Poisson distribution with mean = 300 feet per minute. 0.002 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	PD2 = feet per minute
aux	PD3 = POISSON(300, 0.003)
doc	PD3 = Natural speed of Hiker3 as generated by a Poisson distribution with mean = 300 feet per minute. 0.003 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	PD3 = feet per minute
aux	PD4 = POISSON(300, 0.004)
doc	PD4 = Natural speed of Hiker4 as generated by a Poisson distribution with mean = 300 feet per minute. 0.004 is a seed variable to ensure the same set of random numbers is generated between runs.
unit	PD4 = feet per minute
aux	TotalGap = Gap1+Gap2+Gap3
spec	dt = 1.00000
spec	method = Euler (fixed step)